Тема: Интегрирование по частям

Задание. *Найти неопределенный интеграл.*

$$\int \left(x^3 - 4x + 5 \right) \cos 3x \, dx$$

Решение

Интегрируем по частям три раза:

$$\int \left(x^3 - 4x + 5 \right) \cos 3x \, dx = \begin{vmatrix} u = x^3 - 4x + 5 & du = \left(3x^2 - 4 \right) dx \\ dv = \cos 3x \, dx & v = \frac{1}{3} \sin 3x \end{vmatrix} =$$

$$= \frac{1}{3} \left(x^3 - 4x + 5 \right) \sin 3x - \frac{1}{3} \int \left(3x^2 - 4 \right) \sin 3x \, dx =$$

$$= \frac{1}{3} \left(x^3 - 4x + 5 \right) \sin 3x - \int \left(x^2 - \frac{4}{3} \right) \sin 3x \, dx =$$

$$= \begin{vmatrix} u = x^2 - \frac{4}{3} & du = 2x \, dx \\ dv = \sin 3x \, dx & v = -\frac{1}{3} \cos 3x \end{vmatrix} = \frac{1}{3} \left(x^3 - 4x + 5 \right) \sin 3x - \left(-\frac{1}{3} \left(x^2 - \frac{4}{3} \right) \cos 3x + \frac{2}{3} \int x \cos 3x \, dx \right) =$$

$$= \frac{1}{3} \left(x^3 - 4x + 5 \right) \sin 3x + \frac{1}{3} \left(x^2 - \frac{4}{3} \right) \cos 3x - \frac{2}{3} \int x \cos 3x \, dx =$$

$$= \begin{vmatrix} u = x & du = dx \\ dv = \cos 3x \, dx & v = \frac{1}{3} \sin 3x \end{vmatrix} =$$

$$= \frac{1}{3} \left(x^3 - 4x + 5 \right) \sin 3x + \frac{1}{3} \left(x^2 - \frac{4}{3} \right) \cos 3x - \frac{2}{3} \left(\frac{1}{3} x \sin 3x - \frac{1}{3} \int \sin 3x \, dx \right) =$$

$$= \frac{1}{3} \left(x^3 - 4x + 5 \right) \sin 3x + \frac{1}{3} \left(x^2 - \frac{4}{3} \right) \cos 3x - \frac{2}{3} \left(\frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x \right) + C =$$

$$= \frac{1}{3} \left(x^3 - 4x + 5 \right) \sin 3x + \frac{1}{3} \left(x^2 - \frac{4}{3} \right) \cos 3x - \frac{2}{3} \frac{1}{3} x \sin 3x - \frac{2}{3} \frac{1}{9} \cos 3x + C =$$

$$= \frac{1}{3} \left(x^3 - 4x + 5 \right) \sin 3x + \frac{1}{3} \left(x^2 - \frac{4}{3} \right) \cos 3x - \frac{2}{9} x \sin 3x - \frac{2}{27} \cos 3x + C =$$

$$= \frac{1}{3} \left(x^3 - 4x + 5 - \frac{2}{3} x \right) \sin 3x + \frac{1}{3} \left(x^2 - \frac{4}{3} - \frac{2}{9} \right) \cos 3x + C =$$

$$= \frac{1}{3} \left(x^3 - \frac{14}{3} x + 5 \right) \sin 3x + \frac{1}{3} \left(x^2 - \frac{14}{9} \right) \cos 3x + C.$$

Ответ: $\int \left(x^3 - 4x + 5 \right) \cos 3x \, dx = \frac{1}{3} \left(x^3 - \frac{14}{3} x + 5 \right) \sin 3x + \frac{1}{3} \left(x^2 - \frac{14}{9} \right) \cos 3x + C.$